Multi-Robot Voronoi Coverage under Actuation and Measurement Noise

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I. Introduction

In this paper, we study the problem of multi-robot coverage when the evolution of the robots' positions is uncertain. More precisely, we use the popular Voronoi coverage control approach [1], which is based on the iterative construction of a centroidal Voronoi tessellation, and ask what happens to the control law and the Voronoi tessellation under actuation and measurement noise. We present a first description and analysis of the phenomena (stability of the controller, uncertainty of the Voronoi cells) and show future research directions.

Our problem derives from coordinating a multi-robot system with the objective of coverage. Not uncommon in multi-robot research, the multi-robot setting poses a research question which presents underlying theoretical aspects that relate to multiple other fields. In our case, the spatial uncertainty, the distributed controller and the Voronoi tessellation ask for methods from probabilistic robotics and control of stochastic systems, but also relate to broader disciplines in computer science, such as distributed computing, computational geometry and clustering. The solution lies at the intersection of these fields. In general, a joint understanding of the relevant disciplines is of importance. In particular, multi-robot systems comprise the unique properties of self-organization, interaction and interdependence, which make their study distinct. Once this is recognized, however, whether multi-robot research forms an umbrella term, constitutes its own discipline, or a subset of another discipline seems to be of secondary importance for the successful conduct of research.

II. Problem Formulation

The Voronoi coverage control approach [1] deploys a group of N robots at positions $\mathbf{p}_i \in \mathbb{R}^2$ in a bounded mission space $\Omega \subset \mathbb{R}^2$. Each robot acts as a generator of the Voronoi tessellation $\mathcal{V}$; it iteratively computes and moves toward the centroid $\mathbf{c}_i$ of its Voronoi cell V_i. The Voronoi cell is given at each iteration for the distance function $d\colon\ \Omega^2 \to \mathbb{R}_{\geq 0}$ by

$$V_i = \{\mathbf{q} \in \Omega \mid d(\mathbf{q},\, \mathbf{p}_i) \leq d(\mathbf{q},\, \mathbf{p}_j),\, j \neq i\}\ , \qquad (1)$$

$\forall\, i,\, j \in \{1,\, ...,\, N\}$. Typically, the positions of the generators are assumed to be precisely known, i.e., they are noise-free. The Voronoi tessellation and thus the robot configuration converges to a centroidal Voronoi tessellation (CVT) over time [2]. We now describe the motion of the robots by the discrete time stochastic system model

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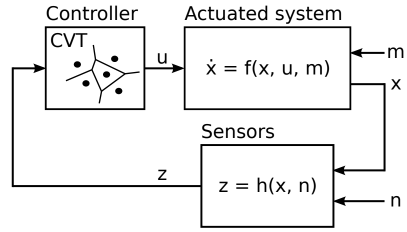

Fig. 1. Voronoi coverage control with actuation and measurement noise.

$$\mathbf{x}_i^{k+1} = f(\mathbf{x}_i^k,\, \mathbf{u}_i^k,\, \mathbf{m}_i^k)\,, \qquad (2)$$

with $\mathbf{x}_i^k$, $\mathbf{x}_i^{k+1} \in \mathcal{X}$ (state space) and $\mathbf{u}_i^k \in \mathcal{U}$ (action space). We include the position $\mathbf{p}_i^k$ in the robot's state vector $\mathbf{x}_i^k$. The motion uncertainty is represented by the actuation noise $\mathbf{m}_i^k$. The robots may have perfect localization that provides them with their exact pose but their localization may also be noisy. In that case, the measurement model of a robot's sensor is given by

$$\mathbf{z}_i^{k+1} = h(\mathbf{x}_i^k,\, \mathbf{n}_i^k)\,, \qquad (3)$$

where $\mathbf{z}_i^{k+1}$ is the measurement and $\mathbf{n}_i^k$ denotes the measurement noise of the sensor. $\mathbf{z}_i^{k+1}$ allows to calculate the state estimate $\hat{\mathbf{x}}_i^{k+1}$ and position estimate $\hat{\mathbf{p}}_i^{k+1}$. The Voronoi cells are then computed based on the estimated positions $\hat{\mathbf{p}}_i$ and $\hat{\mathbf{p}}_j$ instead of the exact positions in (1).

Because of the interdependence between the robots' positions and the Voronoi tessellation, not only the positions $\hat{\mathbf{p}}_i$ but also the Voronoi cells $\hat{V}_i$, their boundaries and the goal points $\hat{\mathbf{c}}_i$ are finally subject to uncertainty. We can formulate the problem as performing Voronoi coverage control under the influence of actuation noise (i.e., the control input is noisy) and under additional measurement noise—given that the state is affected by noise, too. The overall system we are interested in is summarized by Fig. 1.

III. Voronoi Coverage and Noise

In the following study, we use a standard unicycle motion model with state vector $\mathbf{x} = (x,\, y,\, \theta)$ and control input $\mathbf{u} = (v,\, \omega)$, and apply static feedback linearization for its control toward the goal points at the centroids of the Voronoi cells. $N = 3$ unicycles with fix initial states, $\mathbf{x}_1^0 = (1.0,\, 2.0,\, \pi/2)$, $\mathbf{x}_2^0 = (1.0,\, 1.0,\, 0.0)$ and $\mathbf{x}_1^0 = (2.5,\, 1.0,\, 0.0)$ and no initial uncertainty in the states are used for the simulation experiments.

A. Actuation Noise

We first assume the case with no noise in localization, i.e., $\mathbf{n}_i = \mathbf{0}$ and thus $\hat{\mathbf{p}}_i = \mathbf{p}_i$. The actuation noise corresponds to a zero-mean Gaussian distribution $\mathbf{m}_i^k \sim \mathcal{N}(\mathbf{0},\, M_i^k)$.

The basic Voronoi coverage controller [1] for the noise-free ideal case guides each robot along the input vector

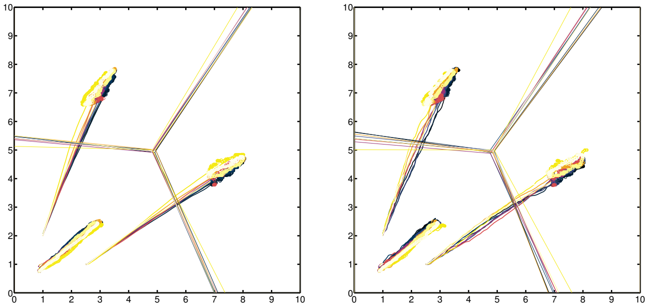

Fig. 2. Convergence to a final CVT configuration under different actuation noise. The trajectories of the robots and the centroids, as well as the final CVTs from 10 runs (color-coded) with three unicycles are shown. The actuation noise is given by the standard deviation in the linear velocity v and the angular velocity ω. Left: $(\sigma_{m_v}, \sigma_{m_\omega}) = (0.4\,\text{m/s}, 40\,\text{deg/s})$. Right: $(\sigma_{m_v}, \sigma_{m_\omega}) = (0.4\,\text{m/s}, 200\,\text{deg/s})$.

$\mathbf{e}_i^k = \mathbf{c}_i^k - \mathbf{p}_i^k$, which makes the robots converge to a CVT. A sufficient condition for global convergence in the presence of actuation noise is to guarantee that $\mathbf{e}_i^k\,\tilde{\mathbf{e}}_i^k > 0$, where $\tilde{\mathbf{e}}_i^k$ is the perturbed input vector (see also [3]). In the more practical case with unicycles, $\tilde{\mathbf{e}}_i^k$ is linked to the control input $\mathbf{u}_i^k$ by a coordinate transformation. If the noise is bounded to a given range (e.g., by physical constraints of the unicycles), the unicycles converge within this uncertainty range. Fig. 2 illustrates that the unicycles manage to converge even under strong actuation noise. Moreover, they reach slightly varying local minima, which indicates that noise can help in avoiding some local minima and reaching other local but more optimal minima.

B. Measurement Noise

We also consider uncertainty in localization, and look at two different ways of localization during Voronoi coverage.

1) Absolute Localization: Each robot estimates its own pose with an Extended Kalman Filter by using measurements of its odometry and a range-and-bearing sensor that measures distance and angle to two fix landmarks in the environment (see Fig. 3 on the left). For the sensor measurements, we assume zero-mean Gaussian noise, i.e., $\mathbf{n}_i^k \sim \mathcal{N}(\mathbf{0}, N_i^k)$. Once localized, each robot communicates its estimated pose to its robot neighbors. As the positions are uncertain, the input vector for the Voronoi coverage controller becomes $\hat{\mathbf{e}}_i^k = \hat{\mathbf{c}}_i^k - \hat{\mathbf{p}}_i^k$ under the measurement noise. If $\mathbf{e}_i^k\,\hat{\mathbf{e}}_i^k > 0$, the robots converge to an estimate $\hat{\mathcal{V}}$ of the CVT. Given zero-mean noise, the mean of the position estimate converges in the same way as the noise-free position $\mathbf{p}_i^k$ of the original deterministic system. The resulting CVT $\hat{\mathcal{V}}$ is a probabilistic Voronoi diagram; probabilistic Voronoi diagrams are studied in [4] under the assumption of bounded uniform noise distributions for the 1D and 2D case. The probabilistic bisectors between the uncertain cells $\hat{V}_i$, in contrast to (1), are described by the probabilities $P(\mathbf{q} \in \hat{V}_i) = P(\mathbf{q} \in \hat{V}_j)$.

2) Relative Localization: Voronoi coverage control does not necessarily require absolute poses and can be executed by the robots knowing only about their relative poses. We let each robot measure the range and bearing relative to its neighbors in order to construct its Voronoi cell $\hat{V}_i$. The input vector can then be written similarly as $\hat{\mathbf{e}}_i^k = \hat{\mathbf{c}}_i^k - \mathbf{p}_i^k$

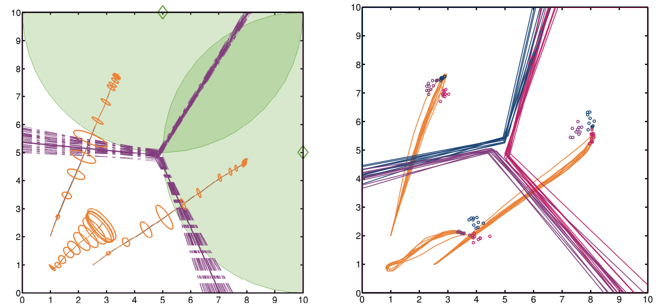

Fig. 3. Noisy absolute and relative localization. Left: The robots localize themselves by odometry and measurements from a range-and-bearing sensor against two fix mapped landmarks at (5.0, 10.0) and (10.0, 5.0) within a range of 5.0 m (green area). The range-and-bearing sensor is modeled by standard deviations in range r and bearing angle ϕ of $(\sigma_{n_r}, \sigma_{n_\phi}) = (0.1\,\text{m}, 1.0\,\text{deg})$. The trajectory of the mean position estimate, the 1σ-confidence ellipses of the position uncertainty and the resulting probabilistic Voronoi diagram with its uncertain bisectors are shown. Right: The robots measure their relative position with a noisy range-and-bearing sensor. Here, the sensor is affected by Gaussian noise ($(\sigma_{n_r}, \sigma_{n_\phi}) = (0.1\,\text{m}, 1.0\,\text{deg})$) with an additional mean offset in the range and bearing angle of $(-0.25\,\text{m}, +5.0\,\text{deg})$. The robot trajectories, final CVTs and centroids from 10 runs with three unicycles are shown. The resulting CVT configuration is not fully covering the environment anymore.

using a global reference frame. As a consequence of the noisy relative measurements, the constructed Voronoi cells are no longer guaranteed to form a seamless covering of the environment and the Voronoi tessellations are usually degenerate (see Fig. 3 on the right).

IV. Conclusions and Future Work

In this paper, we draw the attention to the problem of Voronoi coverage control under actuation and measurement noise. The robots still converge but the uncertainties introduced by noise lead to variations in the deployment process, in the achieved local minima and the final CVT configuration of the multi-robot system.

In our ongoing work, we focus on deriving further guarantees for the controllers as well as the behavior of the tessellations (regarding convergence, robustness, optimality). Heterogeneous systems, where individual robots are effected by noise differently, would introduce different noise distributions. The study of Voronoi tessellations that are overlapping or not fully covering (e.g., CVTs resulting from noisy relative localization), presents another interesting direction. We also see potential applications toward the assessment of the deployment process of a mobile sensor network, e.g., through rating different potential start and final coverage configurations given actuation and measurement noise.

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