Decentralized Control of Networked Systems for Setpoint Tracking

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Decentralized control of networked systems has been widely addressed in the last few years. Main application fields include, for instance, multi–robot systems, distributed sensor networks and interconnected manufacturing equipments. Generally speaking, the aim of decentralized control strategies is implementing local interaction rules to regulate the state of the overall system to some desired configuration. On these lines, mainly investigated coordinated behaviors include aggregation, swarming, formation control, coverage and synchronization.

This work aims at moving a few steps towards the definition of decentralized control strategies for making a multi–robot system implement more complex behaviors. In particular, we will define the complex behaviors to be fulfilled as arbitrarily defined periodic setpoint functions to be followed by each robot.

This objective is obtained partitioning the networked multi–robot systems into two groups: a subset of the robots, referred to as the leaders, may be directly controlled, while the others, namely the followers, are indirectly controlled through the underlying interconnection graph.

As shown in [1], it is possible to model a networked system in such a way that the classical notions of controllability and observability of LTI systems are applicable. In particular, consider a multi–robot system controlled by means of the well known (weighted) consensus protocol, namely

\[ \dot{x} = -L x \]

where \( x \) is the state of the multi–robot system (typically, the vector that collects the positions of the robots themselves) and \( L \) is the Laplacian matrix of the communication graph.

Then, it is possible to partition the Laplacian matrix in such a way that the dynamics of the networked systems can be rewritten as follows:

\[
\begin{cases}
\dot{x}_F &= Ax_F + Bu \\
y &= B^T x_F
\end{cases}
\]

where \( x_F \) is the vector collecting the state of the followers, and \( u \) collects the input injected through the leaders.

In [2] we introduced a decentralized methodology to solve a tracking problem for networked systems in a decentralized manner. In particular, the tracking problem was solved introducing the following control law:

\[ u = F x_F + (\Gamma - \Pi) \xi \]

\[ \Gamma, \Pi \] where obtained from the solution of the well known Francis regulator equations, and the setpoint was defined as a linear combination of a given number of harmonics, generated from the state of an exosystem, namely \( \xi \). The closed loop control scheme is represented in Fig. 1.

The methodology proposed in [2] provides a solution only if the desired setpoint functions are admissible, once the topology of the networked system has been defined. In [3] we introduced a methodology to solve the inverse problem: given the desired setpoint, define the most suitable topology that makes it possible for the networked system to track the setpoint itself.

In order to implement the proposed control scheme, the presence of a complete communication graph among the leaders was assumed in [2]. This assumption was necessary to ensure that all the leaders had access to the full output vector, that was used to build a follower state estimator. However, this assumption is quite restrictive, in realistic operations: in fact, guaranteeing a complete graph among the leaders may excessively constrain the motion of the leaders themselves (if the communication range is limited) or may require huge amount of power (for communicating data at high distances). Therefore, in [4] we introduced a methodology for relaxing this assumption. In particular, in this work we suppose each leader to be able to access one element of the output vector, and we assume only the presence of a connected communication graph among the leaders. Subsequently, we introduce a decentralized estimation procedure, that provides each leader with an estimate of the complete output vector, that is then used for estimating and controlling the state of the followers. It is worth remarking that assuming the presence of a connected graph is much less restrictive than requiring a complete graph: in fact, several strategies can be found in the literature (see e.g. [5]–[9]) for guaranteeing connectivity in a decentralized manner.
It is worth noting that a solution for the regulator equations can be found if the LTI system is controllable. Moreover, a decentralized state observer can be implemented only if the system is observable. In [10] we demonstrated that, with a random assignment of the edge weights, a sufficient condition for ensuring controllability and observability is that of having a connected communication graph.

REFERENCES